Lecture notes in Computational Physics

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Introduction

"the Computers" : female mathematicians who would carry out calculations for observatories and architectural bureaus.

(until ≈ 1930)

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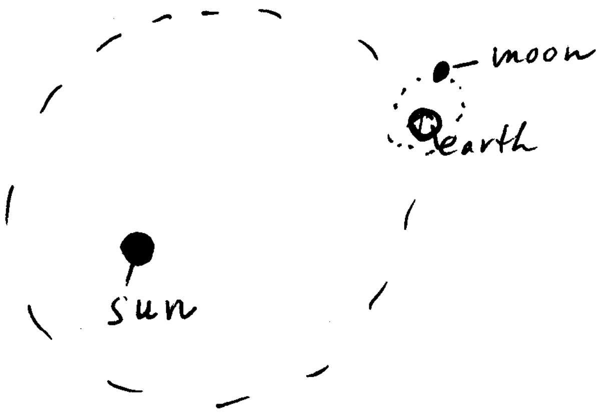
computer, computus (16.th century) } a person who carries out calculations in order to prepare calendars.

$\Longrightarrow$ computational physics has "something to do" with **calendars**.



why calendars?

⇝ important periodic phenomena determining human life

⇝ need for accurate predictions

⇝ religion

data:

1 tropical year = 365,2422 (mean) solar days

= 365 days 5 hours 48 min 46 sec

1 synodic month = 29,530588 days

= 29 days 12 hours 44 min 3,5 sec

$\Rightarrow$ **no** harmonies (Pythagoras, Kepler)

Highlights

A) Maya calendar:

- no decimals or fractions!
- two major cycles: 260 days (luck, religion)
  365 days (sun)

$\Rightarrow$ 52 years cycle (big celebration)

$$\boxed{73 \times 260 = 52 \times 365} = 18980$$

B) Jewish calendar ("moon calendar"):

6×29 days + 6×30 days = 354 days

- Leap month every 3 years, 19 years cycle (Metonic cycle)

C) Egyptian calendar ("solar calendar")

12×30 days + 5 extra days = 365 days

- one leap year every four years (366 days)

D) Gregorian calendar

Gregor XIII. (1582) "Romani Calendarii Explicatio" (1603)

- "re formation" of Julian calendar
- a year comprises 365 days, with a leap year (366 days) every four years, except for multiples of 100, but not multiples of 400, the "computus" becomes quite messy.

"Computus"

- **Metonic cycle:** 235 months ≈ 19 years

quite accurate!

ratio: $\frac{235 \text{ months}}{19 \text{ years}} = 12{,}36842105 \frac{\text{months}}{\text{year}}$

↕

astronomy: $12{,}368267 \frac{\text{months}}{\text{year}}$

- **Basic "calendar formulas":**

Jewish calendar: 98496 years = 1218240 months
= 35 975 351 days

Gregorian calendar: 5 700 000 years = 70 499 183 months
= 2 081 882 250 days

	Jew.	Greg.	Astro.
1 year	365,2468	365,2425	365,2422
1 month	29,530594	29,5305869	29,530588

amazing!

Example 1

"Jewish solar calendar"

a) $X \cdot 365$ days $+ Y \cdot 366$ days
$= 35\,975\,351$ days
(a whole cycle!)

b) $X + Y = 98496$ (years per cycle)

$\Rightarrow X = 74\,188$
$Y = 24\,311$

Example 2

"Gregorian moon calendar"

a) $X \cdot 12$ months $+ Y \cdot 13$ months
$= 70\,499\,183$ months
(a whole cycle)

b) X years $+ Y$ years
$= 5\,700\,000$ years (per cycle)

$\Rightarrow X = 3\,600\,817$
$Y = 2\,099\,183$

A calendar for planet Arcus

1 year = 3,14159265 days

"Metonic cycle": 22 days $\approx$ 7 years

Algorithm 1:

α : real number

$[\alpha]$: integer part of α

- Set: $a_0 = [\alpha]$
  $$\alpha_1 = \frac{1}{\alpha - a_0}$$
  $$\Rightarrow \quad \alpha = a_0 + \frac{1}{\alpha_1}$$
- Iterate: $a_k = [\alpha_k]$
  $$\alpha_{k+1} = (\alpha_k - a_k)^{-1}$$

$$\Rightarrow \quad \alpha = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{\ddots \, a_n + \cfrac{1}{\alpha_{n+1}}}}}$$

continued fraction

if $\frac{1}{\alpha_{n+1}} \stackrel{!}{=} 0$ (n-th Approximation)

$\alpha = 3.141\ 592\ 65358$

0th approx: $a_0 = 3 \Rightarrow \alpha \approx 3$

1st approx: $\alpha_1 = \dfrac{1}{\alpha - a_0} \approx 7.06251335$

bingo!

d.h. $a_1 = 7$

$\Rightarrow \alpha \approx a_0 + \dfrac{1}{a_1} = \dfrac{22}{7}$

2nd approx: $\alpha \approx \dfrac{333}{106}$

3rd approx: $\alpha \approx \dfrac{355}{113}$

4th approx: $\alpha \approx \dfrac{103993}{33102}$

etc.

Similar explanation for accuracy
Of Mother Earth's Metonic cycle !!!

A calendar for planet Arcus:

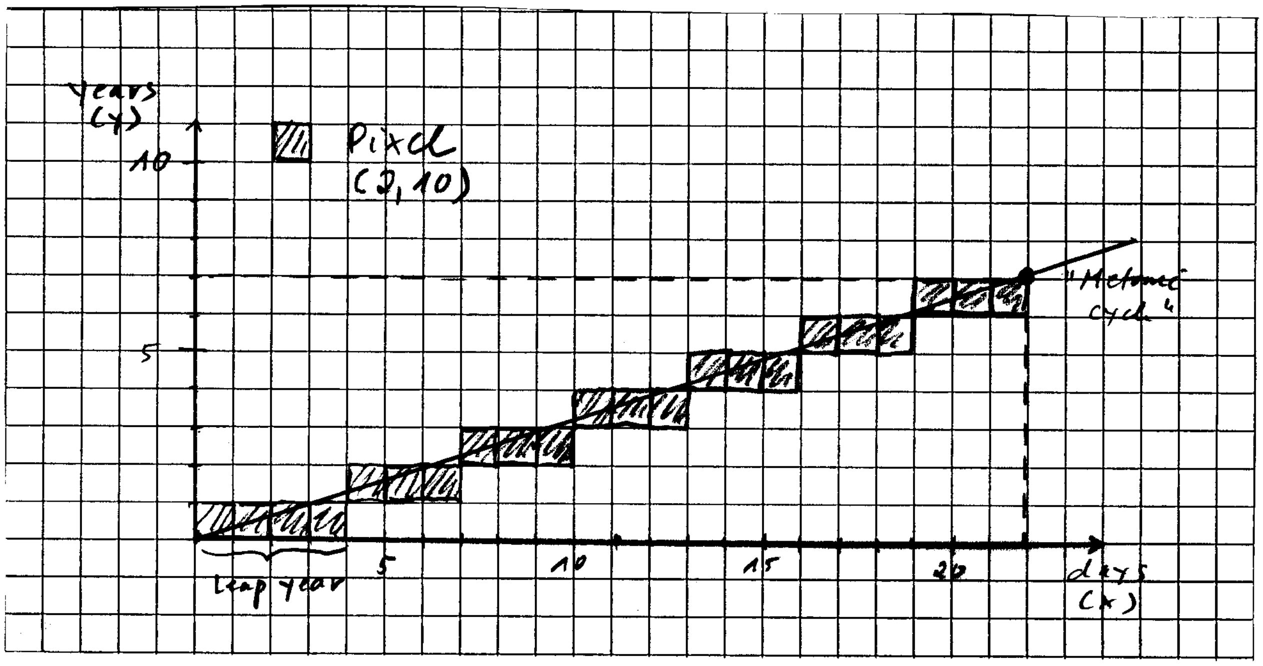
- 7 years cycle comprising 22 days
- leap days, but how?

(6×3 days + 4 days = 22 days)

$\Rightarrow$ systematic procedure ?

Geometric construction

- Idea:



- Bresenham's Algorithm with integer arithmetics

Purpose: Draw line from (x_1, y_1) to (x_2, y_2) on a digital plotter.

Set:

$$\Delta x = x_2 - x_1; \quad \Delta y = y_2 - y_1;$$

$$j = y_1 \; ; \quad \bar{\varepsilon} = \Delta y - \Delta x;$$

$$(\Delta x > \Delta y)$$

Loop:

```
for i from x1 to (x2-1) do
   illuminate (i, j)
   if (ε̄ ≧ 0) then
      j = j + 1
      ε̄ = ε̄ − Δx
   end if
   ε̄ = ε̄ + Δy
end do
```

Arcus' calendar

- draw line from $(0,0)$ to $(22,7)$

$$\Delta x = 22 \qquad \Delta y = 7$$

(x,y)	$\bar{\varepsilon}$	operation
$(0,0)$	-15	draw $(0,0)$; $\bar{\varepsilon}=-8$; new point $(1,0)$
$(1,0)$	-8	draw $(1,0)$; $\bar{\varepsilon}=-1$; new pt. $(2,0)$
$(2,0)$	-1	draw $(2,0)$; $\bar{\varepsilon}=6$; new pt. $(3,0)$
$(3,0)$	6	draw $(3,0)$; $\bar{\varepsilon}=-9$; new pt. $(4,1)$
$(4,1)$	-9	draw $(4,1)$; $\bar{\varepsilon}=-2$; new pt. $(5,1)$
$(5,1)$	....	....
⋮		
$(19,6)$	-14	draw $(19,6)$; $\bar{\varepsilon}=-7$; new pt. $(20,6)$
$(20,6)$	-7	draw $(20,6)$; $\bar{\varepsilon}=0$; new pt. $(21,6)$
$(21,6)$	0	draw $(21,6)$; $\bar{\varepsilon}=-15$; ~~new pt. (22,7)~~ **exit**

- Bresenham's algorithm has originally been invented to draw lines on digital plotters

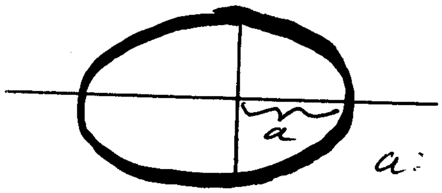
- Contraptions to simplify the "computus" $\Longrightarrow$ | analog / digital computers |

Early computing devices

$\boxed{\text{Kepler's laws}}$ ⇒ Renaissance data mining.

Third law:

$$\left\| \frac{T^2}{a^3} = \text{const.} \right\|$$



a: semimajor axis
T: orbital period

Kepler (1618):

"The proportion between the periodic times of any two planets is precisely one and a half times the proportion of the mean distances".

⇒ Logarithmic law:

$$\frac{T_1^2}{a_1^3} = \frac{T_2^2}{a_2^3} \iff \ln\left(\frac{T_1}{T_2}\right) = \frac{3}{2} \ln\left(\frac{a_1}{a_2}\right)$$

("Mirifici Logarithmorum Canonis Descriptio", Napier 1614)

Schickard (1623) : mechanical computing device.

⇒ simplify the masses of computations necessary to make sense of astronomical data

Kepler was delighted!

M. Mästlin (Kepler's teacher): "it is not seemly for a professor of mathematics to be childishly pleased about shortening the calculations".